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INTERACTION OF A PLANE HYDRAULIC-JUMP WAVE WITH AN ABRUPTLY CHANGING BOTTOM RELIEF IN LOW WATER

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Numerical methods are employed to analyze the structure of a plane hydraulic-jump wave, interacting with an abruptly changing bottom relief, in the two-dimensional low-water approximation. Different modes of this interaction are analyzed in detail.

The field of application of low-water equations is rather wide and includes numerous hydrodynamic aspects of liquid motion in different hydraulic structures and natural basins. The efforts to use a mathematical model of low water to describe real flows to as high an approximation as possible with regard for various physical factors have resulted in the development of a number of complicated models. However, in so doing, elucidation of the basic fundamental properties of the initial simplest model of low water, where the establishment of an analogy with some flows of a compressible gas is the main mathematical achievement, has been left aside. Application of the analogy is effective only in the simplest cases and only for predicting qualitative features of the flows. In two-dimensional nonstationary cases, theoretical study becomes problematic, thus bringing numerical methods to the fore.

The present work is devoted to investigating the interaction between a plane hydraulic-jump wave (the simplest analytical solution of the low-water equations) and a bottom relief in the form of a long deepened canal disposed perpendicular to the direction of jump propagation or in the form of an elevation (a ridge) of the same configuration. This formulation represents a simplified event of interaction that may pertain to some real physical situations when waves propagate in basins with an uneven bottom. The occurrence of various wave disturbances may be attributable to explosions due to the impact of meteor bodies, earthquakes, the motion of ships in narrow canals, and so on.

The investigated system has the form [1]

$$\frac{\partial h}{\partial t}+\frac{\partial hu}{\partial x}+\frac{\partial hv}{\partial y}=0,\quad \frac{\partial hu}{\partial t}+\frac{\partial hu^2}{\partial x}+\frac{\partial huv}{\partial y}=-gh\frac{\partial (h+h_b)}{\partial x},$$
$$\frac{\partial hv}{\partial t}+\frac{\partial huv}{\partial x}+\frac{\partial hv^2}{\partial y}=-gh\frac{\partial (h+h_b)}{\partial y}, \tag{1}$$

where $h(x, y, t)$ is the height of the liquid layer reckoned from the bottom level, specified by the function $h_b(x, y)$; $u(x, y, t)$, $v(x, y, t)$ are the velocity components along the x, y coordinates; g is the acceleration due to gravity. All the linear dimensions here are expressed in cm, the time in sec, and the velocities in cm/sec. The result of solving the system [1] may be extended to other scales according to the following similarity rule: if all the linear dimensions change by a factor of k, the time and the velocities increase by the factor $k^{1/2}$.

At first we will consider the process of passage of a plane hydraulic-jump wave above a relief in the form of a narrow deep canal disposed perpendicular to propagation of the jump. To numerically solve the system of equations (1) with the corresponding initial and boundary conditions, we use the method of [2]. Below we present the results of one of the variants of the calculation for the following initial data:

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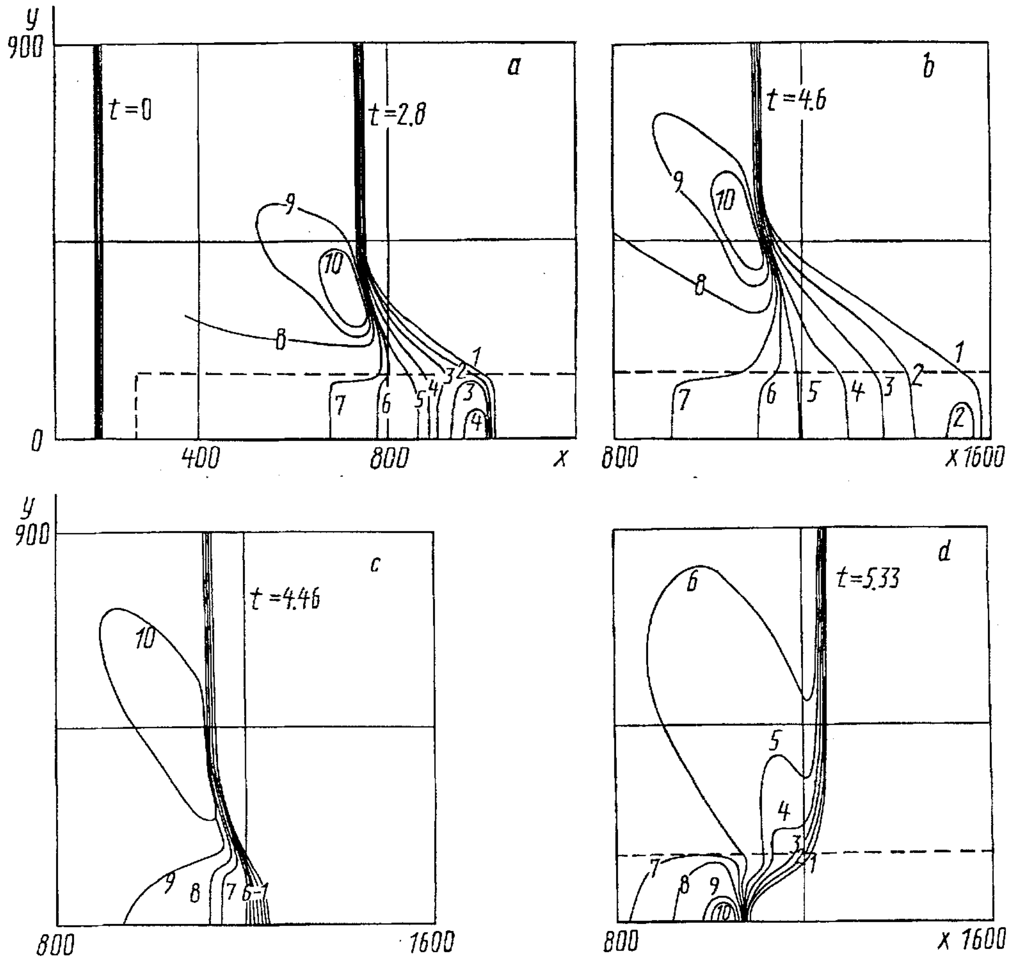

Fig. 1. Structure of the hydraulic-jump wave at t = 2.8, h_c = -60 (a); t = 4.6, h_c = 60 (b); t = 4.46, h_c = -15 (c); t = 5.3, h_c = 20 (d).

$$h = h_0 - h_b,\ h_0 = 30,\ u = u_0 = 0,\ v = v_0 = 0 \quad \text{at} \quad x \geqslant 200;$$
$$h = \alpha h_0 = h_1,\ \alpha = 1,\ 2,\ v = v_1 = 0,$$
$$u = u_1 = (\alpha - 1)(g(\alpha + 1) h_0/2\alpha)^{1/2} \quad \text{at} \quad x < 200.$$

The velocity of the jump is $D = [g(\alpha + 1)\alpha/2h_0]^{1/2}$. The configuration of the bottom is $h_b = h_c = -60$ at $x \geq 300$ and $y < 150$, and outside this space $h_b = 0$. Height gradients of the bottom relief are somewhat overestimated as compared to real configurations. This has been done for the purpose of a more vivid presentation of effects occurring and for the convenience of numerical simulation which does not affect the discussed problems qualitatively. The intensity of the hydraulic-jump wave corresponding to $\alpha = 1.2$, is approximately equal to its limiting height above which, judging from the experiments, its collapse proceeds.

Figure 1a depicts the structure that is realized at t = 2.8. The initial disposition of the hydraulic-jump wave is shown by the vertical lines (t = 0), whereas the space occupied by the cavity of the bottom is marked off by the dashed lines. The level lines correspond to the following heights of the free surface above the undisturbed region, i.e., values equal to $h - h_0 + h_b$: 1, 0.4; 2, 1.1; 3, 1.9; 4, 2.6; 5, 3.3; 6, 4.0; 7, 4.8; 8, 5.5; 9, 6.2; 10, 7.0. The observed effect of the distortion of the front of the hydraulic-jump wave is based on the dependence of the velocity of propagation of the height disturbances on the depth h, which also depends on the bottom configuration h_b. The larger the depth h, the higher the velocity of propagation.Therefore propagating along the deep part of the relief (above the canal on the bottom), the hydraulic-jump wave acquires a higher velocity and the overall front changes its form.

The protruding part of the head jump has an approximately threefold smaller amplitude than the main undisturbed jump. The system of these rectangular jumps connected with an oblique one is followed by a wave of gradual rise of the liquid level (isolines 3-8), which, in its turn, has two characteristic parts. One of them, comparatively flat and narrow, extends above the deepening zone while the other, more pronounced, lies behind the oblique part of the head jump. They join just above a longitudinal cut of the deepening zone (the horizontal dashed line). Isolines 9 and 10 indicate a relative rise of the liquid level behind the region where the oblique jump turns into a straight undisturbed one.

In the course of time, this flow structure does not change qualitatively, and it only enlarges, preserving some similarity and achieving, at large times, a limiting self-similar pattern, in which the width of the canal is no longer the determining factor, and only the cavity of the bottom is of importance. This is confirmed by Fig.1b, where the flow pattern pertains to a later moment of time, i.e., $t = 4.6$. Here the level lines are as follows: 1, 0.3; 2, 1.0; 3, 1.9; 4, 2.7; 5, 3.5; 6, 4.3; 7, 5.1; 8, 5.9; 9, 6.7; 10, 7.5.

As the depth of the canal increases, the amplitude of the jump propagating along the cavity decreases and tends to zero, whereas the velocity of its propagation approaches the finite value $c = (h_0 - h_b)^{1/2}$, i.e., the velocity of propagation of small linear disturbances (an analog of the gasodynamic velocity of sound). In the variant considered, c is larger than the velocity of propagation of the initial jump D. Therefore the distance between it and the beginning of a forerunner will consistently increase.

With decreasing the depth of the canal, the velocity of the forerunner crest also decreases. A value of the bottom cavity in the canal may be chosen (at fixed α, h_0) at which the inequality $c = c^* \leq D$ is fulfilled. This means that at a definite depth of the canal the velocities of the leading part of the forerunner and the main jump become equal and the structure of the flow stabilizes - a stationary flow is realized in the reference system associated with the jumps.

For the case considered above but at $\alpha = 1.3$ and $h_b^* = -15$ (which corresponds to the critical case of establishing a stationary solution) the results of the calculation are represented in Fig.1c. The level lines are as follows: 1, 0.9; 2, 1.8; 3, 2.8; 4, 3.7; 5, 4.6; 6, 5.6; 7, 6.5; 8, 7.4; 9, 8.4; 10, 9.3. Unlike the previous variant, the head jump is not so pronounced and the main part of the flow above the cavity represents a wave of lowering of the level (isolines 4 through 9). The flow did not become quasistationary although it is rather close to this mode.

Generalizing the data of these variants, it may be inferred that in front of the straight jump passing above the bottom cavity a forerunner will be formed at a definite value of deepening of the bottom level and, as a result, a nonstationary self-similar or stationary regime will be realized.

The examples above are the simplest case of manifesting an effect that may conventionally be called the effect of a "narrow deepened canal." It is typical of flows described in the low-water approximation and may have various manifestations, depending on the flow that is realized. It is pertinent to note the most important feature of this phenomenon: local changes of the bottom encompassing a small part of the flow may exert a considerable influence on remote regions and the flow as a whole, i.e., result in total rearrangement of the flow. This effect may be employed with the purpose of weakening different destructive consequences of such geophysical phenomena as tsunami [3].

It is also worth noting that the situation arising is analogous to that occurring in the phenomenon of a "hot layer" [4-12], when shock waves propagating in a compressible gas interact with narrow extended canals of decreased density. Therefore many results of investigating the "hot layer" effect may be extended to the case of a deepened canal. Thus, in describing the change in the flow pattern with changing depth of the canal, we have been concerned with the existence for low-water flows of a criterion, analogous to the Taganov criterion [5, 6], for disturbance of the stationary propagation of a hydraulic-jump wave and emergence of a nonstationary growing forerunner. With the aid of a deepened canal the resistance of a body moving in low water may be decreased just as the resistance of a blunt body moving with a high supersonic velocity is decreased by a hot spike [11, 12]. It should also be noted that as in the case of the gasodynamic "hot layer" effect [12], the "narrow deepened canal" effect in low water occurs not only when the canal is parallel to the direction of the wave velocity but also when it is at some angle to the velocity.

We now describe the case where instead of the bottom deepening there is some local elevation of it (a ridge). For the sake of simplicity, we assume that the configuration of the ridge is analogous to the cavity considered above and that $\alpha = 1.2$, $h_c = 20$. Then, according to the results of calculation, by the moment t = 5.33 the flow depicted in Fig. 1d is realized. The level lines are indicated as: 1, 1.0; 2, 1.9; 3, 2.9; 4, 3.9; 5, 4.9; 6, 5.8; 7, 6.8; 8, 7.8; 9, 8.8; 10, 9.7. Interacting with the ridge, the straight jump forms a configuration consisting of an oblique jump and a region of an abrupt local increase of the liquid level along the axis y = 0 (isolines 9 and 10). At the maximum the level attains a mark that is 3.7 units higher than the liquid level behind the straight jump. It will be recalled that in the case of the straight jump the level increases by 6 units relative to the background value h_0 = 30. This region may be put in correspondence with the Mach leg that appears in the interaction of an oblique shock wave with a wall. Isolines 1-6 pertain to a wave of increase in the level behind an oblique jump having a twofold smaller amplitude than the main jump. In this calculation the initial wave bends inward and no forerunner is formed.

In [13], based on an approximate analysis of the interaction between a jump and a protrusion, the possibility of the existence of a range of parameters where the plane front of a hydraulic-jump wave is disturbed on passing above a local ridge of the bottom and forms an advancing stationary forerunner is shown. The theoretical prerequisites in [13] are based on the assumption, among others, of equality of liquid levels behind the straight jump and in the region above the ridge, i.e., in the region near y = 0. But as follows from the present calculations, this assumption is not fulfilled. The straight jump does not come into direct contact with the near-axis region, whereas an oblique jump with a twofold smaller amplitude abuts this region. Bifurcation regimes [13] might develop at very small amplitudes of the wave-cut step, where the requirement of proximity of liquid levels in all regions is fulfilled. This is indirectly confirmed by the fact that in these cases the requirement $D^* > D$ may be attained, with D^* being the velocity of propagation of the hydraulic jump above the step. This inequality is attributable to the fact that despite a decrease in the velocity of propagation of linear disturbances above the step due to a decrease of the liquid depth by h_b, the relative amplitude of the hydraulic jump α in this region increases somewhat. The some situation develops when the protrusions are very high and almost equal to h_0. In this case the hydraulic jumps above the protrusion are very intense, i.e., $\alpha \approx 5-10$. It should be noted that these conditions are necessary but not sufficient for a forerunner to appear, and they require fulfillment of the same assumption as in [13], i.e., the equality of liquid levels (reckoned relative to the undisturbed state h_0) behind both jumps occurring above the protrusion and beyond it. As is seen, for the regimes with the advancing forerunners to appear in the case of local elevation of the bottom it is necessary to impose rigorous limitations on the type of flow realized. Judging from the calculations conducted a different type of interaction, shown in Fig. 1d is realized in practice.

To sum up, it is pertinent to note that further study of the narrow canal effect, discussed above, seems reasonable. It would be advisable to conduct three-dimensional calculations without the low-water approximation. Experimental checking of the reported results, having the character of a theoretical prediction, is also desirable.

NOTATION

h, water depth; h_b, bottom level relative to some horizontal level; u, velocity in the x direction; v, velocity in the y direction; g, acceleration due to gravity; D, velocity of the undular hydraulic jump; t, time.

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